A NEW MEASUREMENT STRATEGY FOR IN-SITU TESTING OF WALL THERMAL PERFORMANCE*

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ABSTRACT

A new device is proposed for measuring the thermal resistance and the dynamic thermal response of building walls in either the laboratory or the field. The primary departures from past approaches are the control of the time-dependent surface heat fluxes on the specimen and the determination of the surface temperatures as dependent variables.

In this paper the performance characteristics of the device are analyzed by the use of complex thermal admittance. A prototype is being developed based on this analysis. The apparatus is portable and has sophisticated on-line computer control and data analysis through the use of a microprocessor.

Keywords

Building energy conservation, Dynamic thermal performance, Field envelope thermal performance measurements.

INTRODUCTION

The actual thermal performance characteristics of building walls, in-situ, are largely unknown. There is little reason to doubt the underlying theory of heat transport, but variation in construction methods and quality and aging of materials can result in substantial variation in wall thermal performance. Where measurements have been made in actual buildings, the thermal resistance of the walls is often 20% to 30% less than what would be from laboratory measurements and standard calculations. Recommendations for building and planning for energy conservation should be based on what actually happens in buildings rather than on unverified inferences from laboratory measurements and computer models. The methodology for determining the thermal resistance of walls in situ is not well established. Wide variation in instrumentation and technique are reported in the literature [1]. The 20% to 30% discrepancies noted above may be a consequence of inadequate technique. It would be useful, therefore, to develop a reliable method for determining the thermal resistance of building envelope systems through actual field measurement and to validate the method through comparison with established laboratory test methods.

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Existing standard laboratory measurement methods are concerned with steady state heat flow only [2]. The heat capacity of the wall is treated as a complicating side effect. In practice, building walls are subject to time varying heat loads which are dominated by a daily cycle. The heat capacity of walls can be useful in maintaining comfortable temperatures within the building throughout this daily cycle. There are, however, no established standards for measuring the response of building walls to time-dependent heat loads and, thus, it is not possible, at present, to validate experimentally the expected benefits of massive walls.

In this report, we discuss some of the problems encountered in measuring the dynamic thermal characteristics of walls and describe an apparatus which we are designing to solve these problems. The analysis given here should be useful for either laboratory or field measurements.

THE PROBLEM OF MEASURING THERMAL RESISTANCE.

The heat resistance, R, of a wall is the ratio of the temperature difference across the wall to the heat flux through the wall:

$$R = \frac{\Delta T}{\Phi}$$

To determine R, one must measure Φ and ΔT in the same experimental situation. This can be done by maintaining a known temperature difference across the specimen wall and measuring the resulting heat flux, Φ, after the wall and apparatus have reached a steady state. The heat flux may be measured either with "heat-flux sensors" or by inference from the electric power consumed in electric resistance heaters. The two possible methods are shown schematically in Figures 1 and 2.

Heat-flux sensors are small, thin plates of insulating material with a multi-junction thermopile that senses the temperature difference between the two faces of the plate. Heat flux through the plate causes a temperature difference across the plate which, in turn, causes a measurable electric potential difference at the output of the thermopile. The thermal resistance of the heat-flux sensors must be small when compared to that of the wall under test, but it must be large enough to produce a measurable signal from a thermopile containing a practical number of junctions. Heat-flux sensors tend to be light in weight and to have fast response (in the order of seconds). The fast response leads to a serious technical problem: The sensor is very sensitive to transients caused by movement of the air film next to the wall. The response to these can easily be an order of magnitude large than the response to the average heat flux.

On the other hand, measuring the electric power input to electric resistance heaters, is easily done by means of accepted electrical measurement techniques. The heat output is, of course, precisely equal to the electric energy input. Unfortunately, all this heat does not necessarily flow into the wall specimen under test. As shown on Figure 2, the region on the right side of the wall, is subdivided by a box. A thermostatic control is provided to maintain the temperature T_2 equal to T_2'. With these temperatures equal, there should be no heat flow through the auxiliary box, and all the heat output of the electric resistance heaters must then flow into the specimen. The box and the region outside it on the right side of Figure 2 constitute a "guard" that prevents the flux of heat in undesired directions. An apparatus built according to this schematic design is known as a "guarded hot box." Guarded hot boxes are the accepted standard apparatus for measurement the thermal resistance of wall specimens [2].

If one does not control T_2, one must calibrate the hot box by measuring the heat flux through the box in special calibration runs. The calibrated hot box is in use in several laboratories and a standard for it is being developed by a task group of American Society for Testing and Materials, Committee C-16 on Thermal and Cryogenic Insulating Materials.

TIME-DEPENDENT THERMAL MEASUREMENTS

In the standard method for using a guarded hot box, the specimen is subjected to a constant temperature difference. This temperature difference is maintained for a time that is long enough for the whole apparatus to reach a steady state. For walls with massive members, this process can take many hours or even days. Moreover, since walls in buildings are subject to cyclic heat loads, they never achieve the steady state that is created in the test. An analysis of time-dependent thermal response may enable us to better determine the response of building walls to the heat loads which they actually experience. The literature shows a number of approaches to transient behavior. General analytical approaches are described in [3-8] which develop the transient wall application from first principles. An approximate method developed by Mackey and Wright is based on a harmonic analysis characterized by a decrement factor and lag time [9, 10]. These authors also introduced the "sol-air" concept. Modifications designed to achieve greater accuracy are described by Ullah and Longworth [11]. Other formulations of transient behavior are classified as the thermal admittance [12-14] and response factors [15, 16]. Extensions of these approaches to whole-building thermal performance, although not of direct interest to this paper, are treated in several ways in references [17-21] and are mentioned here for completeness. Laboratory-based experimental studies of transient behavior are limited in number; the ones we are aware of are referenced in [22-29] and include the use of analogs in some cases. Experimental studies that include measurements based on actual weather exposure are described in [30-34].

The contents of the above literature have been examined, and are reflected in the following analysis of transient heat transfer in the design of an apparatus for its measurement. In our analysis we treat the wall as a "black-box" and consider only those quantities that can be observed on its exterior surfaces. The observable quantities are:

T_1 inside surface temperature

T_2 outside surface temperature

Φ_1 heat flux into the wall from the building interior

Φ_2 heat flux into the wall from the building exterior

Because we intend to develop an analogy with electric circuit analysis, we have defined heat fluxes as positive when they flow <u>into</u> the "black-box". The temperatures and heat fluxes are functions of time. In the limiting cas of a steady state, the principle of energy conservation requires that $\Phi_2 = -\Phi_1$, but the full time-dependent problem does not have this simplifying restriction. The heat fluxes, Φ_1 and Φ_2, are functions of the driving temperatures, T_1 and T_2, <u>and of the past histories of these driving temperatures</u>. Viewed in complete generality, the problem of characterizing time-dependent wall thermal performance is quite difficult. Fortunately, we can simplify the problem by using the fact that heat transfer and heat capacity are primarily linear processes. The response of the wall to the temperature driving function is a linear superposition of the wall response to a set of basic driving functions. The basic driving functions for which the most experience has been gained in electrical engineering are sinusoidal functions of time. These functions form the basis of the Fourier transform. By measuring the response of a wall to a suitable set of sine-wave driving functions, we can determine enough information about the wall to be able to predict its response to any cyclic drive. If we consider the temperature to be the driving function and the heat fluxes to be the system response, the wall thermal characteristics can be written as a matrix equation:

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = [Y] \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

where [Y] is a 2 x 2 symmetric matrix each element of which is a complex function of frequency. The matrix [Y] of a single homogeneous layer of material is derived in Appendix A. The electric circuit equivalent to a homogenous layer is displayed in Fig. 3, with impedances Z_1, Z_2, Z_m expressed as functions admittances Y_1, Y_2, Y_m. The matrix describing a composite wall can be derived from the matrices describing the component parts. This derivation is presented for a two-layer wall in Appendix B. The two-layer equivalent circuit is shown in Fig. 4. The elements of the wall matrix [Y] have the dimensions of thermal conductance, but are complex. Following the usage of electrical engineering, we call a complex conductance an "admittance," and the [Y] matrix is called the "admittance matrix." The experimental problem is the direct determination of the admittance matrix [Y] from measurements of time-dependent heat flux and temperature. The wall must be driven in such a way that all frequency components of interest are well represented in the driving function and in the response functions. These functions of time are Fourier analyzed and the elements of the admittance matrix are extracted by a regression analysis of the Fourier transforms. This data processing utilizes fast-Fourier-transform programs and least-squares regression programs, both of which are part of the standard library programs in many computer facilities.

DRIVING FUNCTIONS AND RESPONSE FUNCTIONS

The discussion so far as assumed that the wall is driven by imposing a temperature on its exterior, and is tested by measuring the resulting heat-flux response. In the analogous to the situation in electrical circuits, a voltage signal is imposed on the ports of a system and the resulting port currents are treated as the measured response. One can, with equal validity, treat the port currents as driving terms and the port voltages as the system response. For thermal measurements this reversal of roles is seldom done. It seems, however, to have some advantages. For example, the heat output of electrical-resistance heaters is under the direct control of the experimenter whereas the temperatures in the apparatus can be controlled only indirectly through the use of temperature sensors and servo-mechanisms. The servo-mechanism can never be perfect and its imperfection must be understood in some detail, if one is to choose thermal driving functions ccrrectly. Using heat flux drive, on the other hand, allows one to avoid this difficulty.

ENVELOPE THERMAL TEST UNIT

The envelope thermal test unit (ETTU) is a new device being developed at LBL for testing the thermal performance of building walls. It differs from a standard guarded hot box in that it is being designed to be portable and thus to allow the one-site testing of actual building walls. The physical arrangement of the ETTU is shown schematically in Figure 5. The unit consists of two identical "blankets" which are placed in close thermal contact with the wall to be tested. Each blanket consists of a pair of large area electric heaters separated by a low-thermal-mass insulating layer. Embedded in each heater layer is an array of temperature sensors. Heat drive is provided to the primary heaters according to a time-dependent program which covers the interesting frequency spectrum. The secondary heaters are used as guards. Each electric heater is designed to provide a heat output that is uniform over the whole area. The actual heat output is controlled by adjusting the voltage applied to the heater. There are a variety of ways such heaters may be built. Present designs call for the use of printed copper circuits on 0.13 mm (0.005 in) thick mylar®. Although the diagram shows physical separation between the parts of the blankets and the wall, the intent is to have all parts in intimate thermal contact.

The "blankets" are so called because they cover the wall section under test and are slightly flexible, so that they can be made to conform to slight irregularities in the wall surfaces. A microprocessor-controlled data-acquisition system (not shown) is used to drive the system and to record the system temperature responses. By using uniform heat drive, the guard function in the transverse direction is accomplished by restricting analysis to the central region of the blanket. Enough temperature data will be acquired to determine experimentally the size of this central region. Placing the blankets in direct thermal contact with the wall eliminates complications associated with air film and considerably reduces the bulk of the apparatus. The secondary heaters are driven by a servo-control which drives the measured secondary temperature T_s toward the measured primary temperature T_p. This servo-control reduces the temperature gradient across the guard insulation and consequently minimizes the portion of the drive heat that flows into the blanket and does not contribute to driving the wall.

ANALYSIS OF ETTU OPERATION

The several layers of the two blankets of ETTU can be viewed as being more layers of wall, but with an important difference: at four of the interface nodes, the heat input from external sources is <u>certainly</u> <u>not</u> <u>zero</u> as is assumed in Appendix B. Fig. 6 shows the equivalent circuit of the ETTU and the wall under test. The heat fluxes, Φ_3 and Φ_4, are primary heater outputs for outside and inside, respectively. The heat fluxes, Φ_2 and Φ_5, are secondary heater outputs. The servo-control equations are

$$\Phi_2 = G_b(T_3 - T_2)$$

$$\Phi_5 = G_d(T_4 - T_5)$$

where G_b and G_d are servo amplifier "transconductances". (The gain of the servo amplifier is dimensionally a conductance).

T_i is the temperature at node i

Φ_1 and Φ_6, T_1 and T_6 are the inside and outside heat fluxes and ambient temperatures, respectively, which are not under the control of the ETTU control systems.

The data reduction must be done in such a way that one can extract useful information about the wall without making any special assumptions concerning Φ_1, T_1, Φ_1, and T_1. The thermal character of the air films is also unmeasured and it must be shown that it does not affect the data reduction either. The analysis of the servo-control of one blanket is carried out in detail in Appendix C where it is shown that Φ and T are related by the simple formula:

$$\Phi_4 = Y_{fd}T_4$$

where $Y_{fd} = Y_{1d} - Y_{md}$ is determined by calibrating the instrument. An entirely similar derivation yields for the outside blanket:

$$\Phi_3 = Y_{fb}T_3$$

The blanket equivalent circuits in Fig. 6 can be replaced by the much simpler equivalents of these two equations. The resulting matrix equation contains a very tractable 2 x 2 matrix:

$$\begin{bmatrix}\Phi_3\\ \Phi_4\end{bmatrix} = \begin{bmatrix}Y_{1c}+Y_{fb} & -Y_{mc}\\ -Y_{mc} & Y_{2c}+Y_{fd}\end{bmatrix} \cdot \begin{bmatrix}T_3\\ T_4\end{bmatrix}$$

The quantities Φ_3, Φ_4, T_3, and T_4 are all measurable functions of frequency. From these measurements, three functions of frequency can be obtained: $Y'_{1c} = Y_{1c} + Y_{fb}$, $Y'_{2c} = Y_{2c} + Y_{fd}$, and Y_{mc}. The fact that the admittance matrix must be symmetric provides some redundancy in the data that is used to improve the quality of the numerical determinations. The functions Y_{fb} and Y_{fd} are functions of the apparatus only (and not of the wall). They are determined by gathering data on a wall sample of known thermal performance. In the low frequency limit it is easy to see that

$$\lim_{s\to 0} Y_{fx} = 0, \qquad \text{for } x = b, d.$$

This result is not surprising since the insulation and secondary heater are intended to act as a guard. The functions Y_{fb} and Y_{fd} are measures of the imperfection of the guards.

DRIVING SCHEDULES FOR ETTU

In order to determine the functions Y_{ic} from measurement, one must provide non-zero drive fluxes, Φ_3 and Φ_4. Thus, it is seen that applying a drive flux on one side only, the standard procedure used in guarded hot boxes, is sufficient only for the time-independent analysis. Because the arrangement consisting of two blankets is symmetric, it is possible to drive the wall with symmetric ($\Phi_3 = \Phi_4$) signals. The response to the symmetric signal is an effective heat capacity. The response to the antisymmetric signal ($\Phi_3 = -\Phi_4$) is an effective thermal conductance. To ensure that data are collected at all interesting frequencies, we drive the heaters with thermal "white noise". In the present application, thermal white noise is a random signal in which all parts of the frequency spectrum are equally represented. The white-noise signal is generated by switching the heater on and off randomly at a rate that is much higher than the highest frequency at which we collect data. The frequency spectrum of an individual step at time t is.

$$L(f(t)) = \frac{-j}{\omega} e^{-j\omega t}$$

which varies inversely with ω. However a long sequence of such steps at times t_i has a low-frequency spectrum which is uniform in ω. Since the drive signal is generated from on-off switching, proportional control of the primary heaters is not needed. The drive signal is generated by a pseudo-random number generator in the control microprocessor. The shortest time that a heater can be on is currently set at one second, but this can be changed easily in the control program.

The heat-flux drive provided by electric heaters can take on only positive values. If the servo mechanism were perfect, there would be no net heat loss through the blankets heat gain from the primary heaters. This net positive heat gain would result in an unending rise in the temperature of the wall. To avoid this unacceptable situation, two things are done. We build into the servo-mechanism a fixed offset so that the secondary layer is driven to a temperature that differs from the measured primary temperature by a fixed amount, T_0. This offset allows a net heat flux outward through the blankets. With this offset, the wall will have a mean temperature that is somewhat higher than ambient. For more extreme problems we intend to put a layer plastic tubing over the exterior of each blanket and to circulate chilled water through it. This chilled exterior blanket lowers the effective ambient temperature in which the ETTU is operating. The temperature of the chilled water need not be carefully controlled because, as has already been shown, the servo-mechanism is effective at decoupling the wall and test assembly from their immediate environment. In a laboratory, an existing hot box of standard design could be used to maintain a desired mean temperature.

Another approach to maintaining an appropriate offset temperature is to provide low frequency roll-off in the response of the servo amplifier to the temperature of the primary heater, i.e.,

$$\Phi_5 = G_d[R(\omega)\cdot T_4 - T_5]$$

$$\text{where } R(\omega) = 1 \text{ for } \omega \gg \omega_{\text{roll-off}}$$

$$\text{an} \quad R(\omega) \rightarrow 0 \text{ for } \omega \rightarrow 0$$

With reduced gain at very low frequencies, the servo will not follow very slow changes or long-term trends in the temperature of the primary heater. As the heat and temperature in the wall build up, the temperature of the secondary heater will lag behind and a temperature difference, ΔT, will develop automatically. Because there is not a roll-off in the response to the secondary temperature, the servo system becomes a proportional control thermostat at low frequencies. The long-term average value of T_5 must be chosen to be somewhat above ambient so that there is a net flow of heat outward into the room.

To better see how this approach works, consider how it responds to a step temperature input on the primary heater followed by a step temperature rise in the room. Fig. 7 graphically illustrates the system response. The step in T_4 causes heat flux Φ_4 directly and heat flux Φ_5 through the action of the servo. The increase in heat flux Φ_5 cases the rise in temperature T_5. After the initial impulse of heat which causes T_5 to follow the step in T_4, the heat flux Φ_5 decays back to its initial value because of the roll-off in the servo response. The step in temperature T_6 causes a very slight rise in temperature T_5 which, in turn, causes a reduction in heat flux Φ_5 through the servo.

STABILITY OF THE SERVO MECHANISM

We have investigated the servo-mechanism controlling the power drive to the secondary heaters in a computer model. The loop gain was computed as a function of frequency, and Nyquist plots were generated for a wide variety of blanket parameters values. In no case did the real part of the loop gain ever change sign, thus the servo mechanism will always satisfy the Nyquist criterion for stability. Fig. 8 shows a typical Nyquist plot. Changes in the assumed construction materials result in changes of scale but do not affect the shape of the plot. Stability is assumed if this curve does not enclose the point (-1,0).

LATERAL HEAT LEAKS

Lateral heat flow in the sample can cause a problem in interpretation of data from ETTU. This problem has several aspects which we discuss briefly. If there is a uniform transverse heat flow across the test area, there will be a temperature gradient which will be detected by the temperature sensors. In so far as the wall is a linear system, this gradient can be ignored when analyzing heat flow through the wall.

The assumption was made that the gain of the electronic amplifiers was uniform over the frequency span of interest.

A more complicated situation is a net outward (or inward) transverse heat flow at the edges of the test area. Since ETTU measures heat flow at both surfaces, and a temperature profile at both surfaces such a heat flow will be easily detected. The admittance matrix model predicts a perfect heat balance, and no transverse outward flow, so any significant outward transverse heat flow would be detected as a discrepancy with the model. Where there is such a discrepancy, we feel it is not meaningful to speak of an R-value or U-value of the wall at all. Our instrument does not give a false indication in such a situation, rather it gives an indication that the particular section under test must not be characterized as a simple two-port black box. In other words there are no simple parameters which characterize a wall section having large outward transverse heat flows. This situation is not a result of any defects in the measurement methodology or in the instrument design.

Another difficulty which needs consideration are possible inhomogenities in the sample (e.g., a stud and cavity wall). When there are significant inhomogenities it becomes impossible to derive an exact algebraic expression for the admittance. However, our work is directed toward attempting to measure the admittance rather than toward producing precise theoretical predictions for complex constructions.

Our instrument subjects the wall to time dependent heat flux which is uniform over the area under test. Hot-box methods subject a wall specimen to a temperature which is uniform over the area under test. In actual practice walls in buildings experience neither uniform temperature nor uniform heat flux. The situation is like that which occurs in testing electronic devices. The device is driven by a signal source which is either a voltage source or a current source, but in use, the device is driven by a source which has finite impedance. Both voltage source and current source measurement data need to be corrected for the real source impedance. In the case of electronic devices there is a well established formalism for doing this. In the case of dynamic testing of built up wall sections, there is not an established method. We expect that using a weighted average of measured temperatures will be an adequate method, but verification of this expectation awaits the collecting of a reasonably large sample of data.

CALIBRATION

Because the instrument is portable, it is possible to transport it to an existing test laboratory for cross-comparison measurements of static wall response. Dynamic calibration must be done by absolute methods. To make an absolute calibration of the blankets, we use a thick piece of material as a reference block. At high frequencies or for very thick walls, the two sides of a wall decouple from each other and act independently. This phenomena can be seen for a homogeneous single layer wall by taking the limit $\gamma \rightarrow \infty$ in Eq. 22 of Appendix A. In this limit, the mutual admittance approaches zero. Thus we can conclude that the thermal performance of one surface of the reference block can be described by a single complex admittance function of frequency. We denote this function by the symbol, Y_r. By placing the inside blanket against the reference block, we can measure the parallel combination of Y_r and Y_{fd}.

$$Y_{dr} = Y_{fd} + Y_r$$

In a second calibration run, we place the outside blanket against the same surface of the reference block and measure

$$Y_{br} = Y_{fb} + Y_r$$

Finally we can place the inside and outside blankets in direct contact and measure

$$Y_{fo} = Y_{fb} + Y_{fd}$$

In all three of the above runs, the heat fluxes are determined from electrical measurements of the electric power into the heaters and measurements of the physical size of the heater. Temperature is measured with thermocouples. The three runs provide enough information to solve the above equations for the three admittances Y_{fd}, Y_{fb}, and Y_r. For the technique to succeed, we need only assume that these three admittances do not change during the calibration process. We need not assume anything regarding the actual admittance of the reference since it is also determined during the calibration.

DATA ACQUISITION AND CONTROL

Data acquisition and control is done with a commercially available microprocessor-based computer. The signals from the thermocouple temperature sensors are digitized and stored on a floppy disk. Temperature difference signals are computed and used to control the electric power sent to the secondary heaters. The computer generates pseudo-random numbers at regular intervals and uses them to control the primary drive heaters. As data are accumulated, the computer also calculates a Fourier transform of the data streams. The admittance is computed by dividing the transform of the heat-flux function by the transform of the temperature transform of the heat flux function by the transform of the temperature-response function. As the test run progresses, it is possible to determine the admittance vs. frequency function while the test is in progress, thus allowing the operator to suspend further data taking as soon as the admittance appears to have reached a stable value.

For laboratory applications, the measurement method described here can be programmed on existing computer facilities rather than being implemented in a portable microprocessor system.

DISCUSSION

The construction of a prototype ETTU device has been completed. In this model, temperature measurements are made with copper-constantan thermocouples, and the heaters consist of film-deposited copper arrays on a 0.13 mm (0.005 in) thick mylar® polyester substrate. Heaters of this design are used both for the main heat-flux drive and the secondary heater. The prototype design uses bead board for the intra-heater thermal resistance and plywood for structural integrity, later models will use different materials chosen to optimize thermal response of the device. Temperature data are collected with a commercially available data-acquisition system interfaced with the microprocessor computing system.

Initial calibration and performance tests are now being conducted. Preliminary tests using the self-calibration feature have indicated that the servo control algorithm for the secondary heater was unsatisfactory. A new, more complex algorithm that will provide satisfactory servo control is being developed.

CONCLUSIONS

We anticipate that the ETTU, and the associated test procedures that we are developing will provide a feasible and reliable way to measure the dynamic thermal performance of walls in the field, or in the laboratory as an adjunct to existing hot-box apparatus and test procedures.

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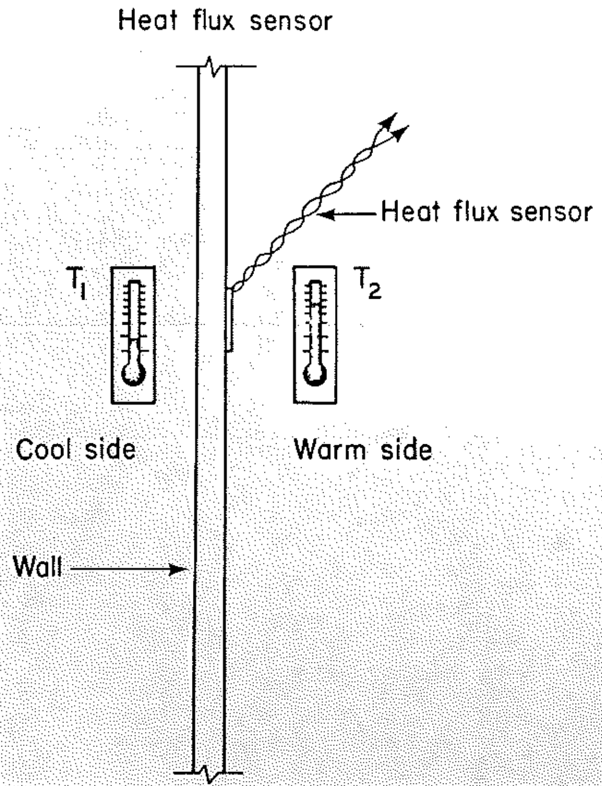


Figure 1. Heat flux sensor method.

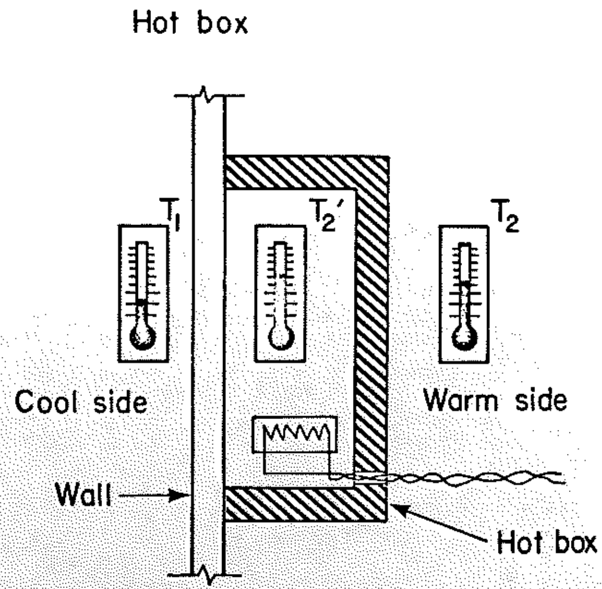
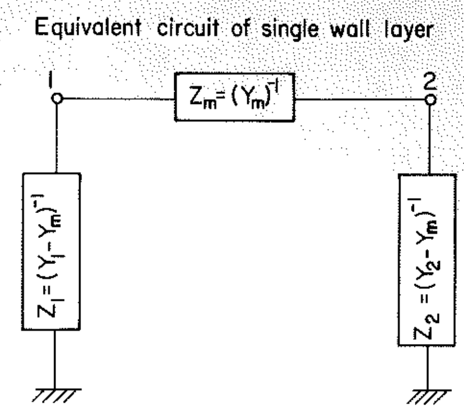


Figure 2. Hot box method.



$Y_1 = Y_2 = \sqrt{s\rho ck}\ \coth\ (d\sqrt{s\rho c/k})$

$Ym = \sqrt{s\rho ck}\ \operatorname{csch}\ (d\sqrt{s\rho ck})$

$s = 2\pi f j$

$j = \sqrt{-1}$

f = drive frequency

ρc = volumetric specific heat

k = thermal conductivity

Figure 3. Single layer equivalent circuit.

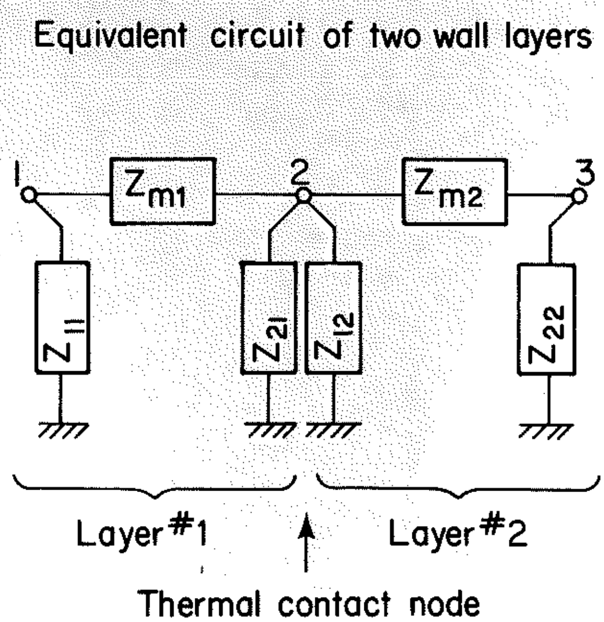


Figure 4. Double layer equivalent circuit.

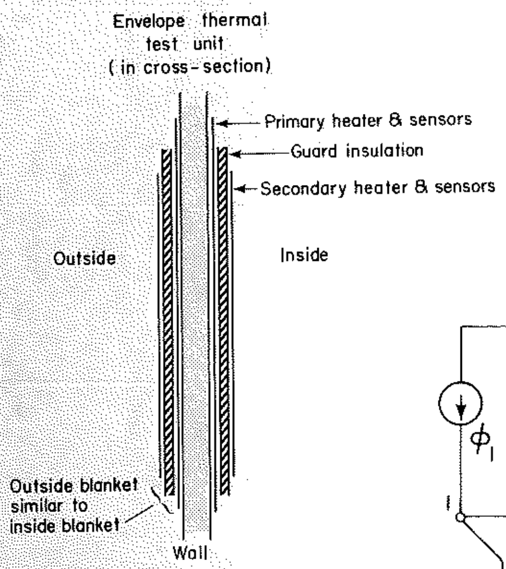


Figure 5. Cross-section of ETTU.

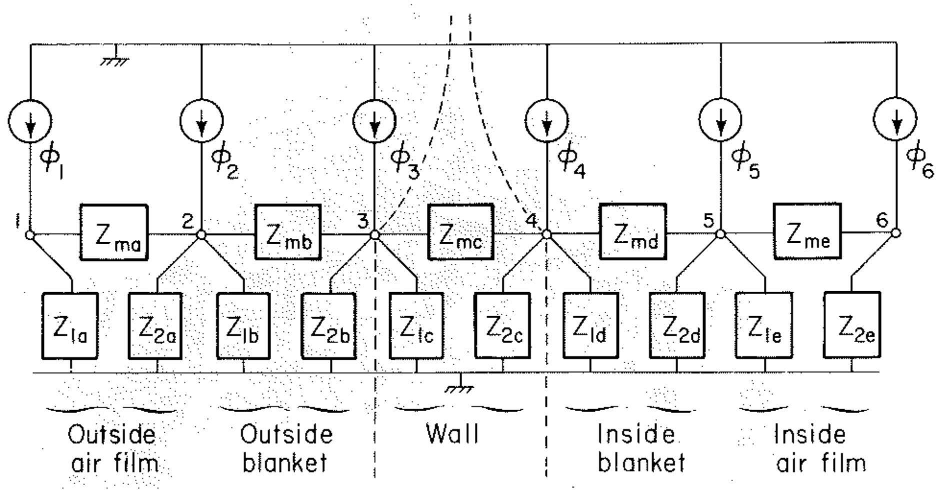


Figure 6. ETTU equivalent circuit. (↓) = Heat flux generator

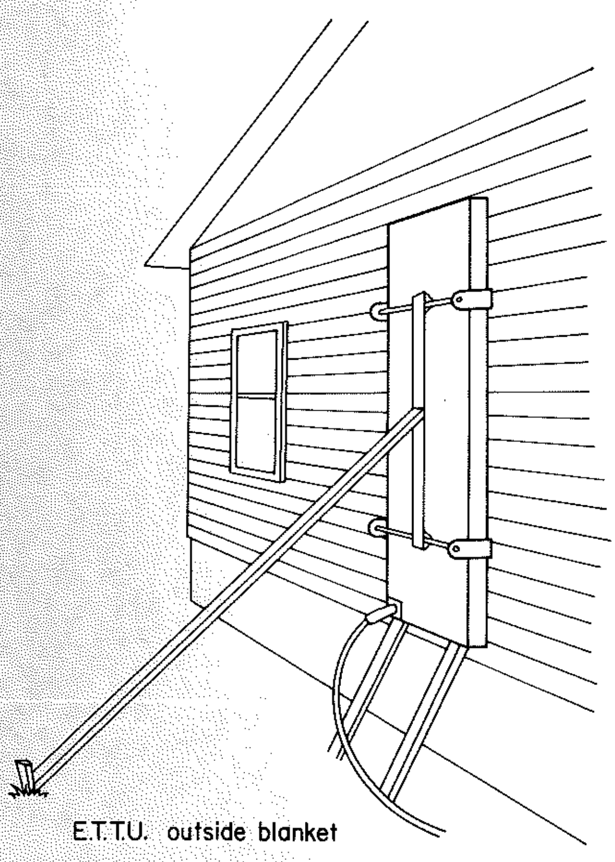


Figure 7. Outside blanket.

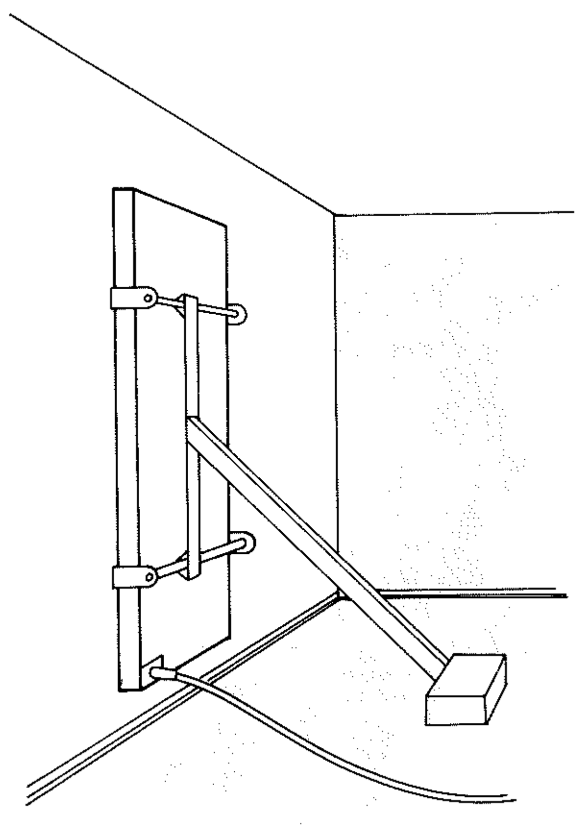


Figure 8. Inside blanket.

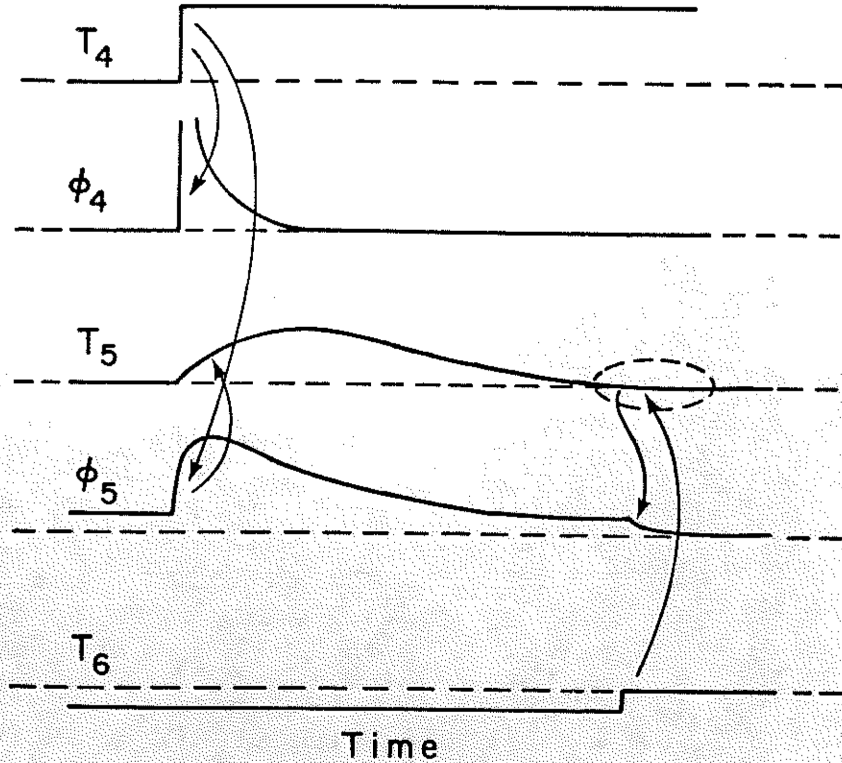


Figure 9. Step response of ETTU.

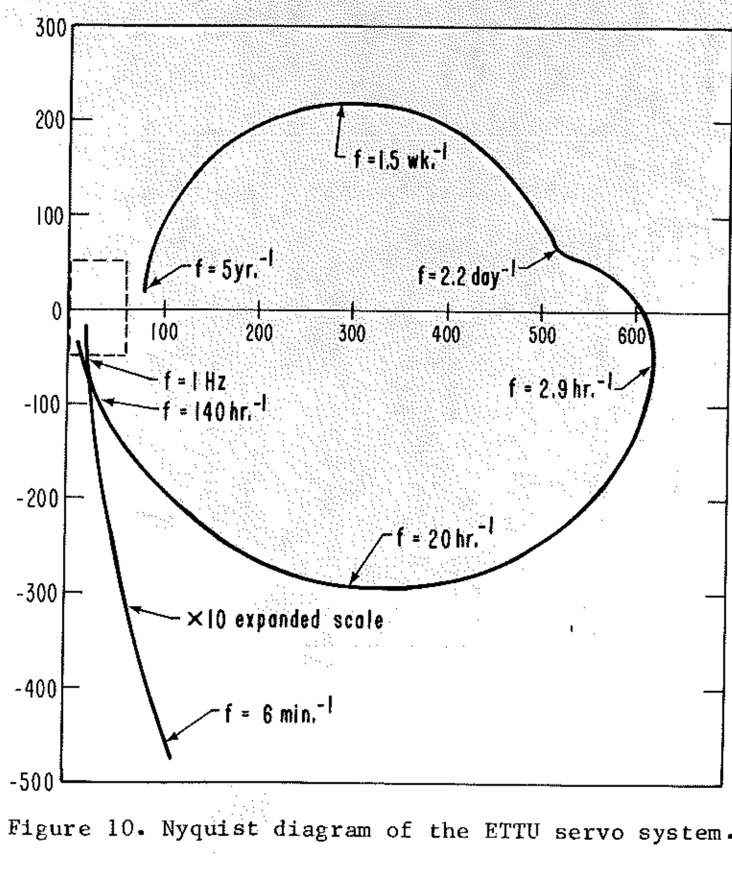


Figure 10. Nyquist diagram of the ETTU servo system.

APPENDIX A. Time Dependent One-Dimensional Heat Flow

Definition of Symbols

T	temperature
Φ	heat flux
k	thermal conductivity
ρc	volumetric heat capacity
x	position
t	time
s	complex frequency coordinate
$\alpha = k/\rho c$	thermal diffusivity
$\beta=\sqrt{s/\alpha}$	frequency-dependent inverse distance
d	thickness of homogeneous slab of material
$\gamma=\beta d$	
Y	thermal admittance, a complex function of frequency, s

The relation between heat flow and temperature gradient is:

$$\Phi = -k\cdot\frac{\partial T}{\partial x}, \tag{A1}$$

where T and Φ are both functions of x and t.

The continuity equation for heat is:

$$\frac{\partial \Phi}{\partial x} = \rho c\cdot\frac{\partial T}{\partial t} \tag{A2}$$

Differentiating Eq. 1 and substituting into Eq. 2 yields:

$$\frac{\partial T}{\partial t} = \alpha\cdot\frac{\partial^2 T}{\partial x^2}, \qquad \text{where } \alpha = k/\rho c \tag{A3}$$

Apply the Laplace-Fourier transform defined by

$$L(f(t)) = \int e^{-st}f(t)dt \tag{A4}$$

and

$$L(\frac{df}{dt}) = s\cdot L(f(t)) \tag{A5}$$

For this transformation to be applicable, the function f(t) must approach zero absolutely at $t = \pm\infty$. This is the case for heat fluxes Φ, but for temperatures we cannot use either the Celsius or the Fahrenheit temperature scales. Rather, we must use the difference between the instantaneous temperature and the long-term average temperature.

In the following equations, Φ and T are functions of x and s rather than x and t. The transform of Eq. (A3) is

$$sT = \alpha\cdot\frac{\partial^2 T}{\partial x^2}. \tag{A6}$$

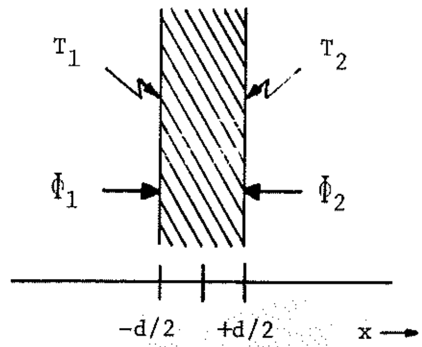
Investigate the general solution

$$T = T_a e^{\beta x} + T_b e^{-\beta x}, \tag{A7}$$

where T_a and T_b are function of s only, and not x. Eq. (A7) is a solution if we require

$$\beta = \sqrt{s/\alpha}. \tag{A8}$$

Now apply these solutions to the particular case of a slab of material whose thickness is d.



We match boundary conditions at

$$x = x_1 = -d/2 \text{ and } x = x_2 = +d/2 \tag{A9}$$

to give the solution for a slab of material of thickness d. At the boundary surfaces we define positive heat flux to be inward into the region defined by $-d/2 < x < d/2$. Thus,

$$\Phi_1 = -k\cdot\frac{\partial T}{\partial x}\bigg|_{x=x_1} \text{ and } \Phi_2 = +k\cdot\frac{\partial T}{\partial x}\bigg|_{x=x_2}, \tag{A10}$$

are the heat fluxes at the boundaries 1 and 2, respectively. For brevity, we define

$$\gamma = \beta d, \tag{A11}$$

and evaluate T and Φ at the boundaries:

$$T_1 = T_a e^{-\gamma/2} + T_b e^{\gamma/2}, \tag{A12}$$

$$T_2 = T_a e^{\gamma/2} + T_b e^{-\gamma/2}, \tag{A13}$$

$$\Phi_1 = -\beta k\cdot(T_a e^{-\gamma/2} - T_b e^{\gamma/2}), \tag{A14}$$

$$\Phi_2 = \beta k\cdot(T_a e^{\gamma/2} - T_b e^{-\gamma/2}). \tag{A15}$$

We form a transfer matrix relating T_1, T_2, Φ_1, and Φ_2. We choose T_1 and T_2 as independent variables and solve for Φ_1 and Φ_2 in terms of T_1 and T_2. Solve first for T_a and T_b. The determinant of the coefficients in Eqs. A12 and A13 is

$$\Delta = -\sinh\gamma.$$

Thus, the general solution for temperature in the layer can be written in terms of (measurable) surface quantities using the relations:

$$T_a = \frac{1}{\Delta}(T_1 e^{-\gamma/2} - T_2 e^{\gamma/2}), \tag{A17}$$

$$T_b = \frac{1}{\Delta}(T_2 e^{-\gamma/2} - T_1 e^{\gamma/2}). \tag{A18}$$

Similarly, the relationship between the surface heat fluxes and temperatures can be written:

$$\Phi_1 = \frac{\beta k}{\sinh\gamma}(T_1\cosh\gamma - T_2), \tag{A19}$$

$$\Phi_2 = \frac{\beta k}{\sinh\gamma}(-T_1 + T_2\cosh\gamma). \tag{A20}$$

Equations 19 and 20 can be combined into a single matrix equation

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \frac{\beta k}{\sinh \gamma} \begin{bmatrix} \cosh \gamma & -1 \\ -1 & \cosh \gamma \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{A21}$$

Dimensionally, the elements of this matrix are all thermal conductances. Since s is complex, these conductances are also complex. In analogy with electrical engineering usage, we call these elements "admittances" and use the letter Y to represent them.

The general form of the Y matrix is

$$[Y] = \begin{bmatrix} Y_1 & -Y_m \\ -Y_m & Y_2 \end{bmatrix} \tag{A22}$$

where Y_1 and Y_2 are the admittances into the first and second surfaces, respectively, and Y_m is the mutual admittance coupling the two surfaces. Notice that there is an explicit minus sign in the definition of Y_m. For a general wall, Y_2 does not equal Y_1, as it does for a monolayer wall.

APPENDIX B. Admittance matrix of a Multi-layer Wall.

The admittance matrix of a multilayer wall can be derived by the repeated application of the following procedure for combining two admittance matrices. For a two-layer system, we use capital Y for the admittance of one layer and miniscule y for the admittance of the other layer. When the two layers are placed in close thermal contact, there are three observable temperatures: T_1 and T_3 are exterior temperatures and T_2 is the temperature at the interface. There are also three heat fluxes in the combined system: Φ_1 and Φ_3 are heat fluxes at the exterior surfaces; Φ_2 is the heat flux into the interface from some external energy resource -- it is not the heat flux through the interface from one layer to the other. Rather, it is the sum of the heat fluxes into the two layers from the infinitesimally thick region between them. To make this more definite we define:

Φ_2' is the heat flux into the capital Y layer,

Φ_2'' is the heat flux into the miniscule y layer.

For the Y layer:

$$\begin{bmatrix} \Phi_1 \\ \Phi_2' \end{bmatrix} = [Y] \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} Y_1 & -Y_m \\ -Y_m & Y_2 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{B1}$$

For the y layer:

$$\begin{bmatrix} \Phi_2'' \\ \Phi_3 \end{bmatrix} = [Y] \cdot \begin{bmatrix} T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} Y_1 & -Y_m \\ -Y_m & Y_2 \end{bmatrix} \cdot \begin{bmatrix} T_2 \\ T_3 \end{bmatrix} \tag{B2}$$

When these two systems are placed in close thermal contact, the combined matrix equation is

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} Y_1 & -Y_m & 0 \\ -Y_m & Y_2+y_1 & -y_m \\ 0 & -y_m & y_2 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \tag{B3}$$

where $\Phi_2 = \Phi_2' + \Phi_2''$.

Since we are considering the two walls to be in close thermal contact with no heat source between them, $\Phi_2 = 0$, necessarily. We use the middle line of the matrix as a scalar equation to eliminate T_2:

$$0 = -Y_m T_1 + (Y_2 + y_1) T_2 - y_m T_3 \tag{B4}$$

$$T_2 = \frac{1}{Y_2 + y_1} (Y_m T_1 + y_m T_3). \tag{B5}$$

Substituting into the equations for Φ_1 and Φ_3:

$$\begin{bmatrix} \Phi_1 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} Y_1 - \dfrac{Y_m^2}{Y_2+y_1} & \dfrac{-Y_m y_m}{Y_2+y_1} \\ \dfrac{-Y_m y_m}{Y_2+y_1} & y_2 - \dfrac{y_m^2}{Y_2+y_1} \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} Y_1' & -Y_m' \\ -Y_m' & Y_2' \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{B6}$$

The Y' matrix is symmetric, as are the initial matrices Y and y.

APPENDIX C. Servo-Control Analysis

Consider the circuit to the right of the right-most dashed line in Fig. 6. Include the feedback system described by the servo control equation:

$$\Phi_5 = G_d (T_4 - T_5). \tag{C1}$$

The system equation becomes:

$$\begin{bmatrix} \Phi_4 \\ G_d (T_4 - T_5) \\ \Phi_6 \end{bmatrix} = \begin{bmatrix} Y_{1d} & -Y_{md} & 0 \\ -Y_{md} & Y_{2d} + Y_{1e} & -Y_{me} \\ 0 & -Y_{me} & Y_{2e} \end{bmatrix} \cdot \begin{bmatrix} T_4 \\ T_5 \\ T_6 \end{bmatrix} \tag{C2}$$

Moving the temperature-dependent terms from the left hand side into the matrix yields the following expression:

$$\begin{bmatrix} \Phi_4 \\ 0 \\ \Phi_6 \end{bmatrix} = \begin{bmatrix} Y_{1d} & -Y_{md} & 0 \\ -Y_{md} - G_d & Y_{2d} + Y_{1e} + G_d & -Y_{me} \\ 0 & -Y_{me} & Y_{2e} \end{bmatrix} \cdot \begin{bmatrix} T_4 \\ T_5 \\ T_6 \end{bmatrix} \tag{C3}$$

For good servo control, G_d will be large and the above matrix will be ill-conditioned. Make a change of variable suggested by the servo control equation to improve the matrix condition:

$$T_5 = T_4 - \frac{1}{G_d} \Phi_5. \tag{C4}$$

This change of variable is accomplished by multiplying the matrix from the right by:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1/G_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{C5}$$

The resulting equation is:

$$\begin{bmatrix} \Phi_4 \\ 0 \\ \Phi_6 \end{bmatrix} = \begin{bmatrix} Y_{1d}-Y_{md} & Y_{md}/G_d & 0 \\ -Y_{md}+Y_{2d}+Y_{1e} & -1 - \dfrac{Y_{2d}+Y_{1c}}{G_d} & -Y_{me} \\ -Y_{me} & Y_{me}/G_d & Y_{2e} \end{bmatrix} \cdot \begin{bmatrix} T_4 \\ \Phi_5 \\ T_6 \end{bmatrix} \tag{C6}$$

In the limit of large gain ($G_d \rightarrow \infty$), this becomes:

$$\Phi_4 = (Y_{1d} - Y_{md})T_4 = Y_{fd}T_4, \tag{C7}$$

and a set of subsidiary equations which allow the determination of Φ_5 and Φ_6. The important result is that the relation between Φ_4 and T_4 is decoupled from the rest of the relations. The blanket surface at node 4 behaves as a single thermal admittance that is independent of the values of T_6 and Φ_6 and of the admittance matrix of the air film. The admittance Y_{fd} is frequency dependent and must be determined in calibration procedures. A similar algebraic development for the outside blanket leads to the relation

$$\Phi_3 = (Y_{2b} - Y_{mb})T_3 = Y_{fb}T_3 \tag{C8}$$

The admittance Y sub fb must also be determined in calibration procedures.